Surface Shape Reconstruction of Transparent Objects using Structured Light [Times Roman 18 bold centered]

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Abstract [Times Roman 10 bold]: Reconstruction of a transparent phase object based on the varying periodicity of an incident sinusoidal intensity pattern is proposed and verified by using a *cm*-thick glass cylinder. [Times Roman 10]

Keywords [Times Roman 10 bold]: holography, structured light, 3D imaging [Times Roman 10]

1. Introduction [Times Roman 10 bold; Roman numbers]

Over the past several decades, 3D imaging of reflective objects have matured and been commercialized. However, reconstruction of transparent/translucent objects, important in machine vision, remain a challenge. Such objects have complicated transmission/reflection mechanisms; therefore, it can be challenging to infer their 3D morphologies from captured images directly. Besides physical and chemical intrusive methods, holographic interferometry, direct ray measurement, distortion recovery method, polarization imaging, etc. have been proposed. However, the limited detection zone and depth-of-field of holographic techniques may limit its application to relatively small objects such as cell specimens [1]. Other methods rely on complex computer vision algorithms to obtain 3D point clouds and corresponding normal vectors from multiple shots [2], but polyhedral objects still pose problems. Polarization imaging estimates surface shapes by analyzing the polarization state of the light [3]. [Times Roman 10]

In this paper, a simple non-interferometric incoherent light propagation model is introduced to perform 3D profiling of transparent objects with typical thicknesses in the order of *mm* to *cm* by analyzing the distorted captured image behind the object. A two-dimensional (2D) cosine fringe is used as the incident reference image, whose periodicity is markedly altered by the shape of the object. As a proof-of-principle, thick plano-convex blocks of glass are used. By monitoring the local change in the period, the surface profile is simulated and optimized to achieve minimal error with experimental data to determine the final morphology. Our proposed method is simple, robust, straightforward, single-shot, and can be used with coherent or incoherent illumination. Moreover, the implementation of this technique on arbitrary transparent objects is theoretically feasible and promising.

2. Theory

Based on the laws of refraction, the light paths through the transparent object can be computed, as shown in Fig. 1. For simplicity, the flat surface of the object above is assumed to be the incident surface and perpendicular to the incident rays. A camera is used to image the plane closest to the highest point of the object, which is at a height H from the incident plane. The normals N_A and N_B of any two points A and B on the curved surface are indicated in Fig. 1, and rays passing through them (in blue) are deflected to AA' and BB', respectively. Also, in Fig. 1, BB'' represents the undeviated ray passing through point B with a height of h_B from the flat (incident) face. For an incident angle α and refraction angle β,

$$\beta = \alpha + \varphi, \ tan(\varphi) = \frac{B'B''}{BB''}. \quad \text{[centered; equation number right adjusted]} \qquad (1)$$

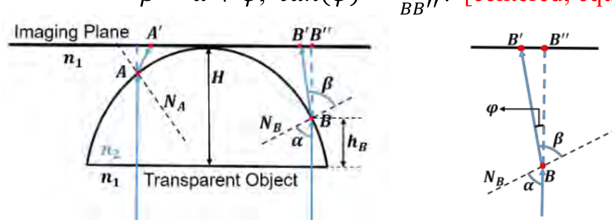


Figure 1 [Times Roman 9 bold]: Illustration of the ray propagation model. Left: a top view of parallel light rays passing through a transparent object. Right: the enlarged view of triangle $BB'B''$, defining the angle φ. [Times Roman 9]

Using Snell's law, the relationship between α and β can be written as

$$n_2 sin(\alpha) = n_1 sin(\beta), \qquad (2)$$

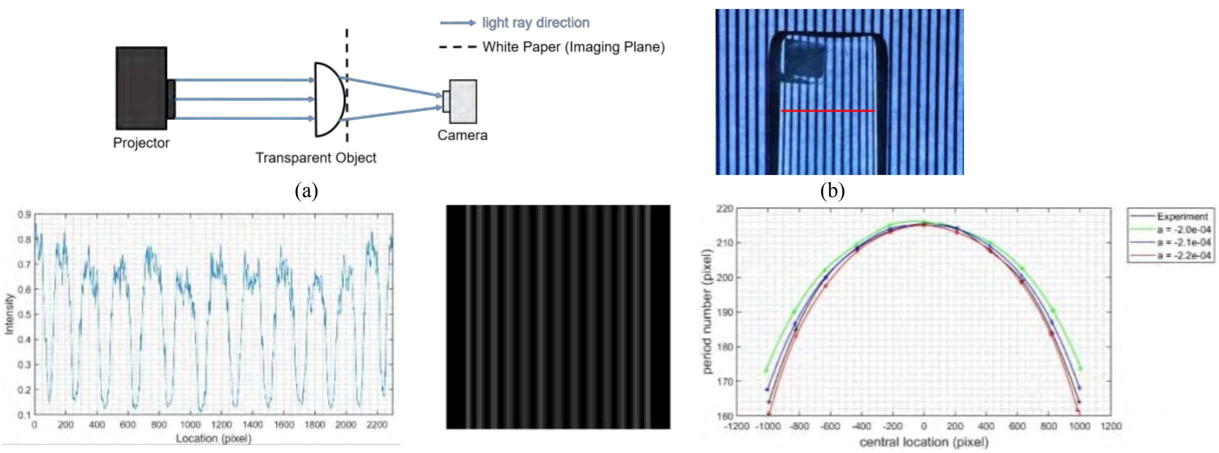
where $n_1(= 1)$ and n_2 are the refractive indices of air and the test object, respectively. For simplicity, we assume that for $\alpha > \theta_c$, where θ_c is the critical angle given by $\theta_c = arcsin(n_1/n_2) = arcsin(1/n_2)$, the light encountering total internal reflection is lost and not recorded by the camera. Combining Eqs. (1) and (2), the displacement $B'B''$ of the ray incident at point B at a height h_B can be derived as:

$$B'B'' = (H - h_B) \times tan\left[arcsin\left(n_2 sin(\alpha)\right) - \alpha\right]. \tag{3}$$

Based on the ray propagation model in Eq. (3), the path of any reference pattern passing through a transparent object can be calculated. In our experiments, a cosine intensity profile is taken as the reference pattern since the deformation of its periodicity can be readily determined.

3. Experimental Setup and Results

As an illustrative example, the surface reconstruction of part of a glass cylinder is presented. As shown in Fig. 2(a), a projector is used to generate the sinusoidal reference image, and a camera (Canon EOS 800D, 6000×4000 pixels) is focused on the exit plane (calibrated using a piece of white paper with an picture) which is imaged to record the deformed pattern. It is ensured that the object size is smaller than the projected range of illumination, as shown in Fig. 2(b). A typical plot of the captured intensity variation is plotted in Fig. 2(c). The period as a function of position is derived and plotted, along with simulated periods from typical deformed images (Fig. 2(d)) from passing through plano-convex objects with their curved surfaces modeled as $y = ax^2$ with different parabolicities a. The three simulated cases shown in Fig. 2(e) are the ones closest to the experimental data (superposed). The most optimum value of the parabolicity a was determined to be $a = -2.16 \times 10^{-4}$. From this, the maximum height of the curved surface is determined to be $8.982576\ mm$, which is in excellent agreement with the actual height of $9\ mm$.



(a) (b) (c) (d) (e)

Figure 2: Illustration of experimental process: (a) the top schematic view of setup; (b) captured image showing deformation within the boundary of the object; (c) the intensity distribution of one extracted horizontal line, shown in red in (b), of the deformed pattern; (d) one of deformed images from a simulated object with a parabolic curved surface; (e) measured period of deformed cosine pattern plotted as a function of location, along with calculated periods of three parabolic cylinders with different curvatures. The abscissa (which we call $\boldsymbol{x}$) is shifted to center the curves at $\boldsymbol{x} = \boldsymbol{0}$.

4. Conclusion

The proposed procedure can be extended to determine more complicated surface profiles through determining the local radii of curvatures by comparing the local periods to those obtained from a databank of fitting curves, using deep learning techniques. Our technique should be scalable and adaptable to mapping *mm* or sub-*mm* sized phase objects.

5. References

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