

Algebra

Easy (25 Points): Find the slope intercept form of the line that goes through the point (3,2) with slope of 2019.

Medium (50 Points): Which is biggest 2^{256} or 4^{129} or 8^{85} ?

Hard (100 Points): A right circular cone's radius is cut in half while the height is tripled. What is the new volume if the old volume was 400 meters squared? Note that the volume of a right circular cone with radius r and height h is given by the formula: $\frac{1}{3}\pi r^2 h$

Counting

Easy (25 Points)

James is picking out an outfit to wear for the cold weather. He needs to wear either his red coat or his blue coat. His red coat is not quite as warm as his blue coat, so if James wears the red coat, he will also need to wear one of his 4 scarves. If he wears his blue coat, he cannot wear a scarf because it would make him too hot. No matter which coat he chooses, James will also need to wear one of his 3 hats to keep his ears warm. How many different winter outfits can James choose from?

Medium (50 Points)

There are ten restaurants near Sam's house, all of them serve breakfast, lunch, and dinner. One day, Sam decides to go to a different restaurant near his house for each of his three meals. He is not too picky and doesn't care about which restaurant he visits for any particular meal, so he will let his picky brother Alex, who has decided to come along, choose the order when it is time to leave. Sam's only preference is that he doesn't want to eat at Marion's Pizza and Dewey's Pizza on the same day because he gets tired of pizza easily. How many ways can Sam choose three out of the ten restaurants to visit?

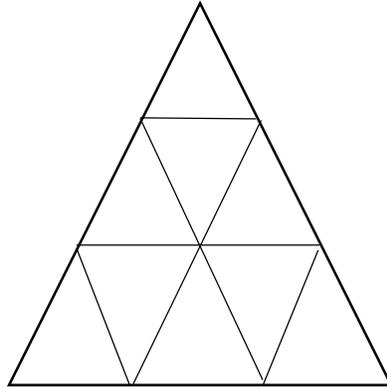
Hard (100 Points)

A certain combination lock has 3 places to input a digit from 0 to 4 (inclusive). However, something is broken inside the lock's internal mechanism, so some 3-digit codes cannot be inputted. A code can be inputted if the second digit is greater than or equal to the first digit, and the third digit is greater than or equal to the second digit. For example: 000, 123, 144, 224 are valid, but 010, 432, 231 do not work. How many valid codes are there?

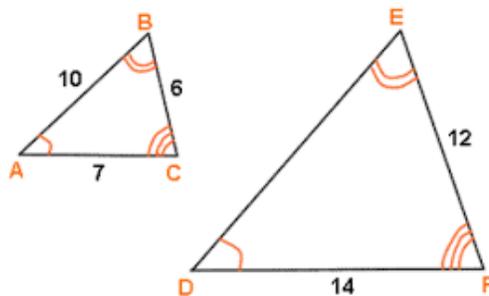
Geometry

Easy (25 points) Find the area of the triangle below comprised of 9 congruent triangles each with a base of 4 and height of 7.

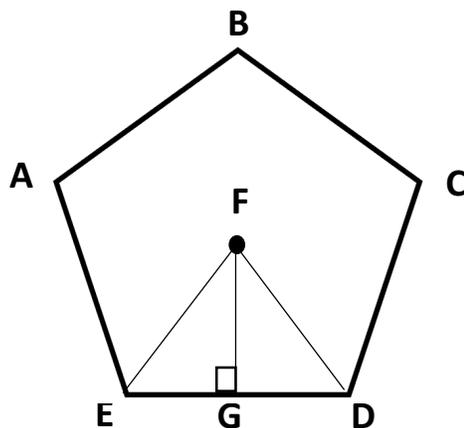
Not to scale:



Medium (50 points) Determine if the pair of polygons is similar. If so, find DE and explain why the polygons are similar. If not, explain why.

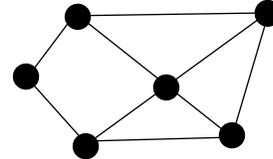


Hard (100 points) Find the area of the regular pentagon with center F shown below. It is given that $EF=10$.



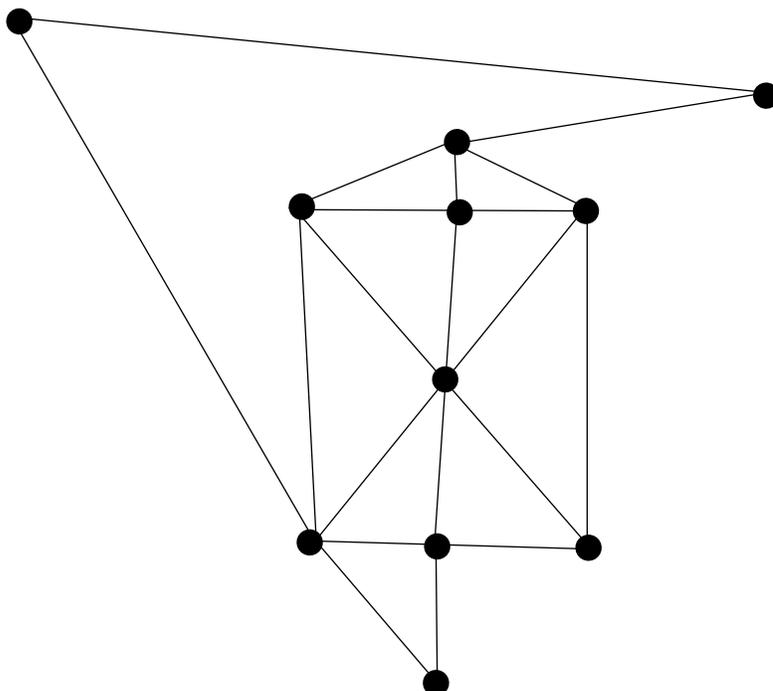
Graph Theory

A graph is a mathematical object that is composed of a set of vertices and a set of edges. A graph can be represented visually as shown. The vertices are depicted by dots. An edge must join two vertices; edges are shown as lines (or arcs) between two dots. The graph shown has six vertices and nine edges. An edge between two vertices represents the existence of a relationship between those two vertices. Two vertices that do not have this relationship are not joined by an edge. Two vertices are called adjacent if there is an edge that joins them. Here is an example of a visual representation of a graph:



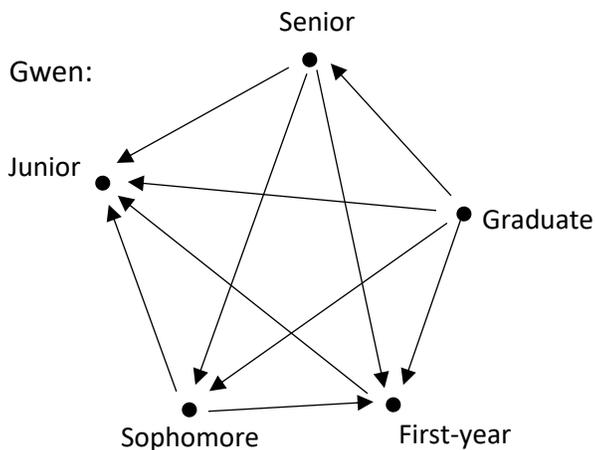
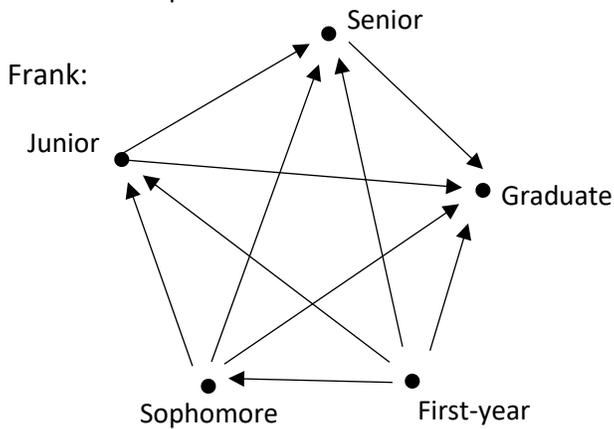
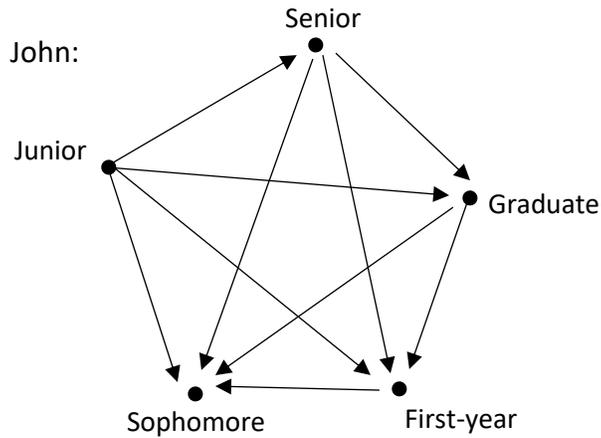
Easy: Lance is having a few friends over for sandwiches, namely Amanda, Bob, Carmen, Dave, Ellie, and Fernando. Lance has prepared some snacks prior to their arrival, but only one sandwich of a kind. The sandwiches are a PB & J (p), a BLT (b), a Grilled Cheese (g), a Tuna Fish (t), a Club (c), a Sub (s), and a Reuben (r). Lance asks his friends which of these sandwiches they like, and their answers are: Amanda (b, t); Bob (b, c, s); Carmen (t, c, s); Dave (p, b, g, t); Ellie (b, t, c, s); Fernando (b, t, c). Lance will have whatever sandwich is leftover. Represent this situation as a graph and determine if it is possible for all of Lance's friend to have a sandwich they like.

Medium: A Hamiltonian cycle must traverse the edges to go through every vertex exactly once and return back to the starting vertex. It can use an edge only once but does not need to use every edge. Find a Hamiltonian cycle for the graph below.

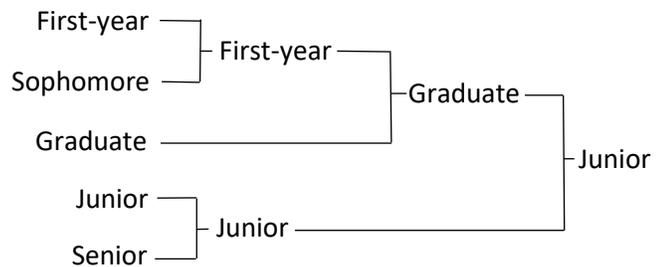


Hard: A directed graph is one in which each edge has an orientation from one vertex to another.

Three students: Frank, John, and Gwen are given the task of choosing a student to serve on a committee. Each class (First-year, Sophomore, Junior, Senior, Graduate) nominates one student. The directed graphs below are the preferences for Frank, John, and Gwen. An example of how to interpret these directed graphs is: John and Frank prefer first-years to sophomores while Gwen prefers sophomores to first-years.



John, wanting Juniors to win, proposes the process of elimination using a bracket style and suggests the order below for pairing eliminations. John, Frank, and Gwen propose their preferences between the pair. Majority wins. The winner then faces a new pairing of whichever class remains. This process repeats until there is one student remaining.



Question:

Which order for pairs would lead to the senior winning?

History of Math

Easy (25 pts): When Carl Gauss was in grade school, his teacher asked the class to find the sum of every whole number between 1 and 100, expecting the question to take a long time to solve. Gauss astonished his teacher by giving the correct answer in seconds by using a clever shortcut. But what if all the even numbers were subtracted, instead of added? That is, find the sum:

$$1 - 2 + 3 - 4 + \dots + 97 - 98 + 99 - 100$$

Medium (50 pts): Leonhard Euler (pronounced OIL-er) was an 18th century mathematician who was responsible for a lot of the equations and notation that we still use today. For example, he introduced $f(x)$ to represent a function, discovered the number $e \approx 2.718$, and popularized the use of the symbol π . During the working years of his life, he published 500 writings, averaging a total of 800 pages written during a year. If half of his writings had exactly 40 pages each, and the other half had exactly 120 pages each, how many years did he spend writing?

Hard (100 pts): For a long time, the ancient Greeks didn't believe that there were any irrational numbers (numbers that can't be represented as the ratio of two whole numbers). The story goes that the Greek mathematician Hippasus was the first to prove that $\sqrt{2}$ is an irrational number, but the other Greeks didn't like that, so they threw him off a boat. Nowadays we know that irrational numbers are perfectly valid, so we can solve equations like the following:

$$x^2 + \sqrt{2}x = 4$$

Find and simplify the values of x which satisfy the above equation.

Logic Problems:

Easy (25 pts):

Three identical-looking umbrellas are sitting upright in a stand. Each have tags to identify their corresponding owner. Assuming the owners do not check the tags, what percentage chance is there that only two of the three people will walk off with their own umbrella?

Medium (50 pts):

A store was robbed, and four suspects: A, B, C, and D were brought in for questioning. Based on the interrogations of them the following facts were established:

1. If A is guilty, then B was an accomplice.
2. If B is guilty, then either C was an accomplice or A is innocent.
3. If D is innocent, then A is guilty and C is innocent.
4. If D is guilty, so is A.

Among the four suspects, which ones are innocent and guilty?

Hard (100 pts):

Alice is lost in the Forest of Forgetfulness and loses track of what day it is. She comes across two brothers: Tweedledee and Tweedledum. One of them always tells lies on Mondays, Tuesdays, and Wednesdays but always tells the truth every other day of the week. The other brother always lies on Thursday, Friday, and Saturday but always tells the truth every other day of the week. Alice notices a toy between the two brothers and tries to determine which brother it belongs to. She asks one of the brothers, "Who owns this toy?". The brother replied, "The true owner of the toy is lying today." What are the chances that the speaker owns the toy?

Number Theory

Easy (25 pts)

What prime numbers divide 111^2 ?

Medium (50 pts)

What is $\gcd(2019, 2019^2 + 1)$?

(*gcd means the greatest common divisor.*)

Hard (100 pts)

Suppose p is a positive prime number such that $p^2 - 1$ is not a multiple of 24. What is/are the possible value/values of p ?

Probability

Easy (25 points) Two regular dice with 6 sides are rolled. What is the probability that the sum of the two dice is even? Simplify your answer.

Medium (50 points) What is the probability that a randomly selected 6 letters from the letters S, A, E, N, K, would spell SNAKES? Simplify your answer.

Hard (100 points) What is the probability of getting 53 Sundays in a leap year?

Surprise

Easy (25 pts)

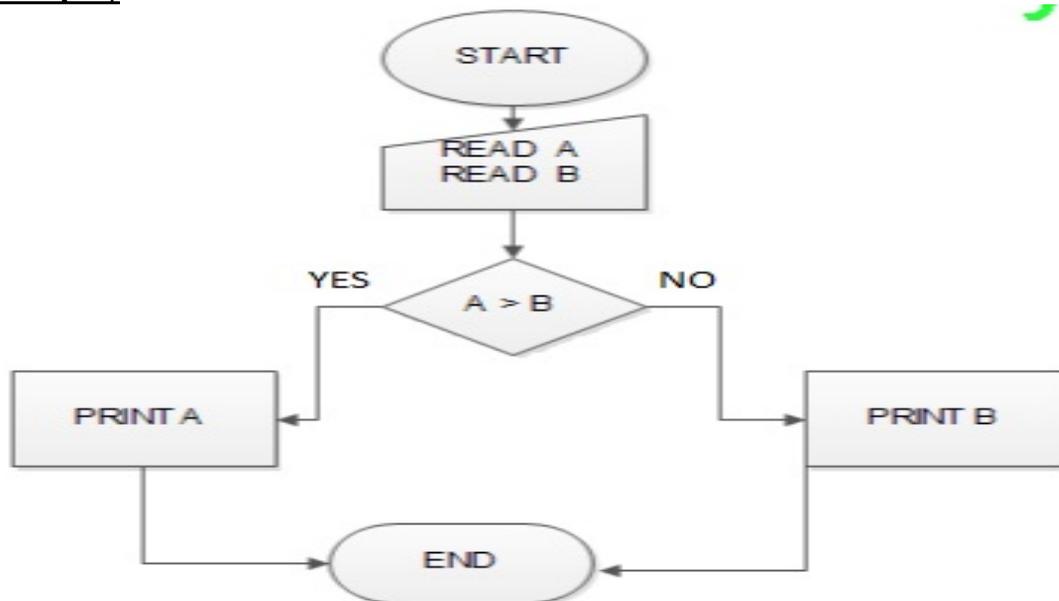
A two-person game has the following rules. Begin with the number **zero**. **Player 1** selects an integer **n** such that $1 \leq n \leq 10$; and adds the integer to zero to determine a new number. **Player 2** selects an integer **n** such that $1 \leq n \leq 10$; and adds that integer to the previous number to determine a new number. The players alternate turns, each adding an integer **n** such that $1 \leq n \leq 10$; to the previous number. The player who reaches **100** is the winner.

Determine if there is a strategy that guarantees a player victory. Would you want to go first or second to make the strategy work?

Medium (50 pts)

What is the sum of the angles of a polygon of **n** sides?

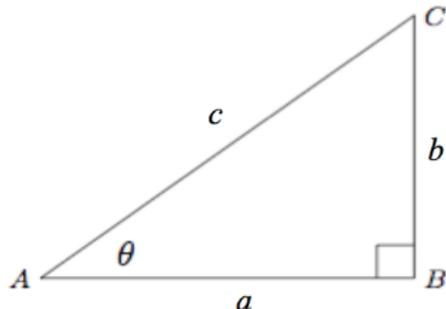
Hard (100 pts)



Suppose **A** and **B** are two random integers between **1** and **n**. Observe the algorithm above. Let 'Event A' be defined as the event which results from printing **A** and let 'Event B' be defined as the event which results from printing **B**. What is the probability of 'Event A'?

Trigonometry

Easy (25 pts) Given triangle ABC and $a=7$, $b=x$, and $c=x-1$. Find x .



Medium (50 pts) A man is standing 900 feet from the base of a building, and looking up at an angle of 30 degrees to the top of a 40-foot statue that stand on the roof of the building. Find the height of the building.

Hard (100 pts) The area of triangle CDB is equal to 50 units². Given the figure below, what is the length of AB? Leave answer in radical form.

